Published August 4, 2015

A Mathematical Equation to Calculate Linear Distance of Cyclic Horizons in Vertisols

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Abstract

Deep and wide shrinkage cracks and slickensides are developed in Vertisols due to the presence of shrink-swell minerals like smectites. As a result of swelling of these minerals, vertical and horizontal shear stress pushes the clay mass upward forming parabolic thrust cones as cyclic horizons. A mathematical equation has been proposed for measuring the linear distance of these cyclic horizons taking into account the depth of occurrence of slickensides. This equation is an improvised, but useful, method to identify locations of microdepressions and microknolls required for better management of Vertisols.

Vertisols and their intergrades occupy a large portion of the world's arable lands. They cover 790 million acres (320 mha) and are spread over 76 countries. Although they are extensive, they remain underutilized for food production. Most Vertisols are considered highly productive, but are difficult to manage due to typical shrink-swell minerals (e.g., smectite), limited moisture contents for tillage, and high energy demand for cultivation. The importance and impact of morphological properties of these soils have been recognized for both agricultural and nonagricultural uses (Soil Surv. Staff, 1951; Simonson, 1954; Bhattacharjee et al., 1977; Blokhuis, 1982; Wilding and Tessier, 1988; Williams et al., 1996). Vertisols develop deep and wide shrinkage cracks during summer (Fig. 1, *I* and *II*). These cracks close as the soil rewets due to upthrusting that forces a swelling clay mass to slip over another resulting in formation of slickensides (Bhattacharjee et al., 1977; Blokhuis, 1982; Wilding and Tessier, 1988). The cyclic horizons repeat in the subsoil, the size of which depends on the length of cycle. One-half of the linear distance (LD) of the cycle is a measurement of the lateral dimension of a cyclic horizon (Hole, 1961; Johnson, 1963). Despite the volume of literature on Vertisols, no relatively simple method has yet been proposed that can determine the relationship between depth of occurrence of slickensides and linear distance of cyclic horizons. With this in mind an attempt has been made to develop a mathematical equation to calculate the linear distance of cyclic horizons in these soils. This information could be useful in the management of Vertisols.

Materials and Methods

The LD has been calculated assuming a parabolic path of the cyclic horizons as shown by Bhattacharjee et al. (1977) and Wilding and Tessier (1988) (Fig. 1). To develop the mathematical equation, the standard equation for a parabola [($y^2 = 4ax$) where a = focus of the parabola] has been used. The concept of cyclic horizons of Vertisols in relationship to their parabolic path centers

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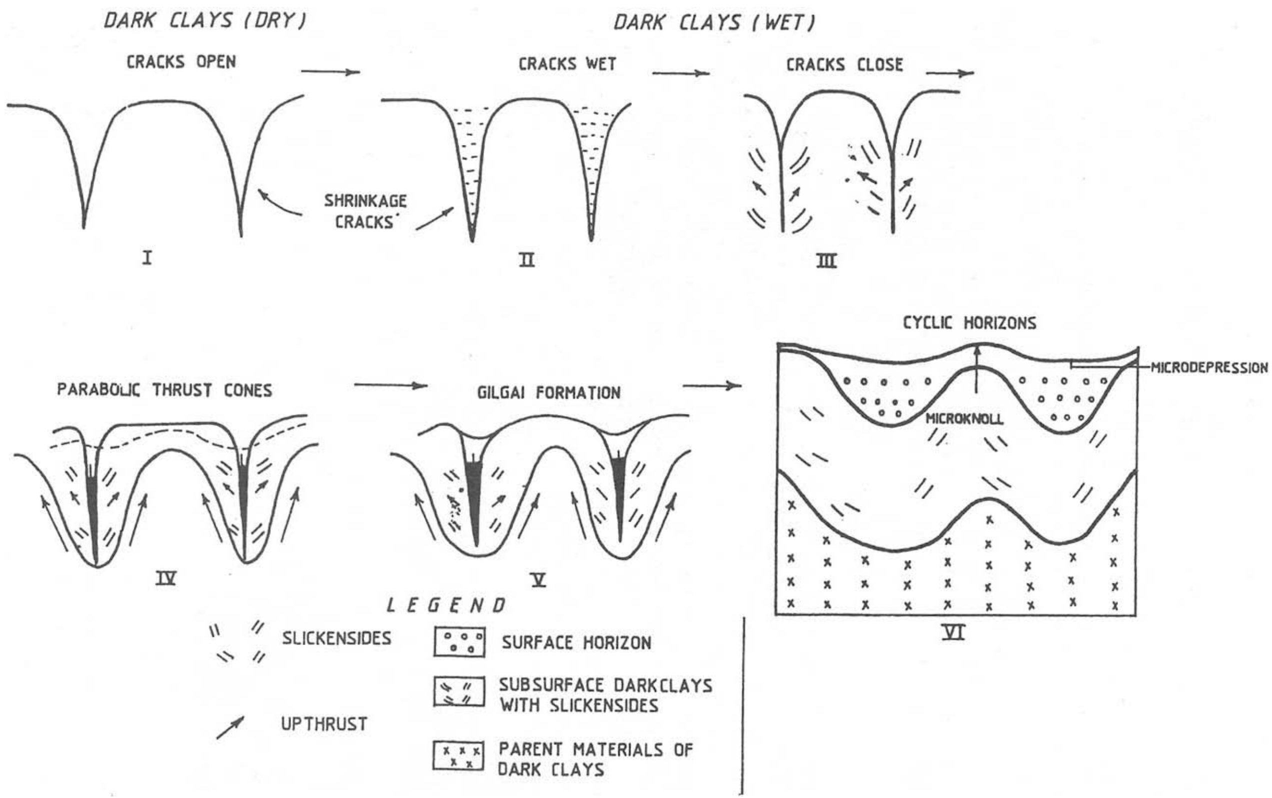

Fig. 1. Different stages (I–VI) of Vertisol (dark clays) development.

around two basic assumptions. The first assumption is the depth of first occurrence of slickensides (b) coincides with the focus of the parabola. The second assumption is an apparent constancy of b within a cyclic horizon.

Results and Discussion

In Fig. 2, let $OS = a$ and $SK = b$. Therefore, $OK = a + b$ = maximum depth of cyclic horizon that corresponds to maximum depth of slickensides and

$$(KN)^2 = 4(OS)(OK) = 4(a)(a + b) \text{ or,}$$

$$KN = 2\,[a(a + b)]^{1/2}.$$

Here $LD = MN = 2KN = 4[a(a + b)]^{1/2}$. [A]

Calculation of LD using Eq. [A] requires the values of a and b. The value of b corresponds to the depth of the first occurrence of slickensides that can be identified in the field (Vadivelu and Challa, 1985). But for the value of a, the profile has to be examined exactly in the center of cyclic horizon. This is difficult since subsurface features like cyclic horizons can not be identified from the surface. This is even more difficult in arable lands where microknolls and microdepressions are obliterated due to cultivation. Therefore, if the profile is examined away from the center of the cyclic horizon, calculation of linear distance will be difficult in absence of the value of a. We propose the following equation to determine a in the profile represented by $CDEF$ (Fig. 2).

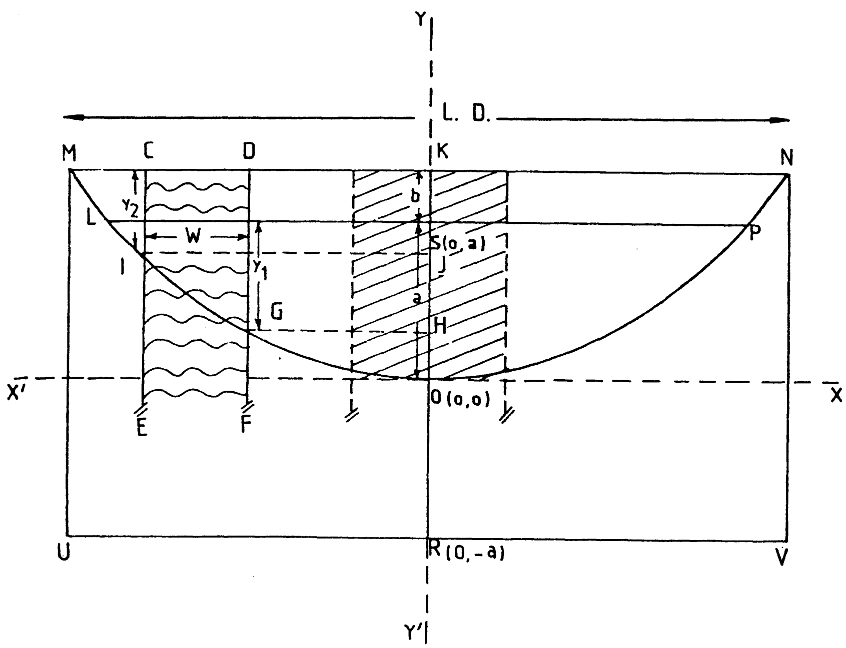

Fig. 2. The parabolic path of cyclic pedon where PL, S, UV, O, and KR are latus rectum, focus, directrix, vertex and axis of the parabola ($KN = KM$, $OS = OR$, $KN = NV$, $KM = MU$, $OJ - OH = y_1 - y_2 = HJ$, $OJ + OH = y_1 + y_2$, $OJ{\bullet}OH = y_1y_2$).

Here $(GH)^2 = 4(OH)(OS)$ or,
$GH = 2(OH)^{1/2}(OS)^{1/2}$ and
$(IJ)^2 = 4(OJ)(OS)$ or,
$IJ = 2(OJ)^{1/2}(OS)^{1/2}$.
Therefore, $IJ - GH = 2OS^{1/2}[(OJ)^{1/2} - (OH)^{1/2}]$
$= 2a^{1/2}[(OJ)^{1/2} - (OH)^{1/2}]$.
Again $IJ - GH = w$(width of the pit); therefore
$w = 2a^{1/2}[(OJ)^{1/2} - (OH)^{1/2}]$ and
$a = w^2/4(OJ + OH - 2(OJ)^{1/2}(OH)^{1/2}$
$= w^2/4[y_1 + y_2 - 2(y_1 y_2)^{1/2}]$.

Hence $LD \quad = 4[w^2/4(y_1 + y_2 - (y_1 y_2)^{1/2})\{w^2/4(y_1 + y_2 - 2(y_1 y_2)^{1/2}) + b\}]^{1/2}$.

For a normal soil pit of 100-cm width (w),

$$LD\text{ (cm)} = 400/2[1/(y_1 + y_2 - 2\,(y_1 y_2)^{1/2})\,[\{10^4/4(y_1 + y_2 - 2\,(y_1 y_2)^{1/2})\} + b]]^{1/2}$$
$$= 200[1/Y^2\{(10^4/4Y) + b\}]^{1/2}$$
$$= 200/Y(2500 + bY)^{1/2} \quad [B]$$

where, $Y = y_1 + y_2 - 2(y_1 y_2)^{1/2}$ and y_1 and y_2 are vertical distances (cm) from the first occurrence of slickensides to the intersecting points of cyclic pedon in such a way that $y_1 > y_2$ and b is the depth of first occurrence of slickensides (Fig. 2). It always is possible to find out the values of y_1 and y_2 if the profile is examined on either side of the center of the cyclic horizon. And since b is known, the LD value can easily be determined following Eq. [B].

Validation of Equation

Equation [A] was tested with available field data from three locations on representative Benchmark Vertisols (Murthy et al., 1982) under different climatic conditions (Bhattacharjee et al., 1977). The test results follow:

Test 1

Climate, semiarid; location, district Ahmednagar, Maharashtra State, India; benchmark series, Umbraj; classification, Typic Haplustert

b = first occurrence of slickensides = 0.71 m
$a + b$ = total depth to the bottom of parabola = 1.50 m
$\therefore a = 1.50 - 0.71 = 0.79$ m

Therefore $LD = 2(KN)$
$= 4[a(a + b)]^{1/2}$
$= 4[0.79(a + b)]^{1/2}$
$= 4.[0.79(1.50)]^{1/2}$
$= 4(1.185)^{1/2}$
$= 4.35$ m.

Field studies indicate medium-size cyclic pedons with a LD measurement varying between 3.00 to 3.82 m in length (Bhattacharjee et al., 1977).

Test 2

Climate, semiarid; location, district Ahmednagar, Maharashtra State, India; benchmark series, Otur; classification, Typic Haplustert;

$$
\begin{aligned}
b &= \text{first occurrence of slickensides} = 0.55 \text{ m} \\
a + b &= \text{total depth to the bottom parabola} = 1.50 \text{ m} \\
\therefore a &= 1.50 - 0.55 = 0.95 \text{ m}
\end{aligned}
$$

$$
\begin{aligned}
\text{Therefore } LD &= 2(KN) \\
&= 4[a(a+b)]^{1/2} \\
&= 4[0.95(a+b)]^{1/2} \\
&= 4[0.95(1.50)]^{1/2} \\
&= 4(1.19)^{1/2} \\
&= 4.77 \text{ m.}
\end{aligned}
$$

Field studies indicate medium-size cyclic pedons with a LD measurement varying between 3.70 to 4.00 m in length (Bhattacharjee et al., 1977).

Test 3

Climate, subhumid; location, district Nagpur, Maharashtra State, India; benchmark series, Linga; classification, Typic Haplustert;

$$
\begin{aligned}
b &= \text{first occurrence of slickensides} = 0.18 \text{ m} \\
a + b &= \text{total depth to the bottom parabola} = 1.20 \text{ m} \\
a &= 1.20 - 0.18 = 1.02 \text{ m.}
\end{aligned}
$$

$$
\begin{aligned}
\text{Therefore } LD &= 2(KN) \\
&= 4[a(a+b)]^{1/2} \\
&= 4[1.02(a+b)]^{1/2} \\
&= 4[1.02(1.20)]^{1/2} \\
&= 4.42 \text{ m.}
\end{aligned}
$$

Field studies indicate medium-size cyclic pedons with a LD measurement varying between 2.00 to 3.00 m in length (Bhattacharjee et al., 1977).

Degree of Accuracy of the Equation. Using the higher values from the field observations as the standard, the degree of accuracy has been estimated as 86, 81, and 53% respectively for three test sites. It has been reported the depth of slickensides increases in areas where rainfall is high (Vadivelu and Challa, 1988). It appears that with greater depth of slickensides, the bottom of the cyclic pedon (*O* in Fig. 2) will be pushed further down. This will alter the shape of the pathway and may cause it not to be exactly parabolic. This may be the reason why this equation shows a low degree of accuracy in subhumid to humid areas. For these areas another equation needs to be developed. However, in the locations we evaluated arid to semiarid climates, this equation can be used with a range of accuracy between 81 to 86%. Only Eq. [A] was tested because of the absence of y_1 and y_2 values. Exact field values of these two variables will further increase the degree of accuracy of Eq. [B].

Since Vertisols occur dominantly in semiaridic environments, the proposed equation will be effective on most Vertisols.

Usefulness of the Proposed Equation. The movement of Vertisols by swelling and shrinking are directly related to microknolls and microdepressions of the cyclic horizons developed within the soils. Thus the linear frequency of microknolls and microdepressions of gilgai microrelief determine the shallow depth of the crest of the cycle at microdepressions (Bhattacharyya et al., 1977). It has been observed that parabolic thrust cones are formed with microdepressions more than 1 m deep in "cat clays" in parts of the Black Land Prairie areas in Texas, USA (Williams and Touchet, 1988). These authors observed slickensides beginning at a depth of 65 cm on microknolls and extending downward to a depth of 183 cm in microdepression. The horizontal distance between microknolls and microdepressions was measured at about 2 m. The amplitude and wave length of the cycles are controlled by a number of factors. The lower depth of each cyclic pedon (*O* in Fig. 2) is determined by climate, and nature and amount of clay (William et al., 1996; Vadivelu and Challa, 1985). Observations in a number of countries indicate the wave length of the cycle ranges from about 2 to 5 m horizontally and the amplitude varies from 75 to 125 cm vertically. In India, amplitudes of 150 cm has been reported (Bhattacharjee et al., 1977). To evaluate such subsoil variability to determine the length of cyclic pedons (LD) it is necessary to have trenches at least 10 m long with depths of 2 m or more (Bhattacharjee et al., 1977; Williams et al., 1996). This field work is time consuming, laborious and expensive. The equations presented here may have potential to eliminate some field work by determining the LD with the help of three variables (Fig. 2), namely y_1 (the length of slickensided zone on the right-hand side of the profile wall from the depth of occurrence of slickensides), y_2 (the length of surface and the slickensided zone on left-hand side of profile wall) and b (depth of occurrence of slickensides). These values can easily by obtained by soil scientists involved in soil survey and mapping.

Many acres of Vertisols are used for pastures. Cracks in these soils may be wide enough to cause dangerous footing for animals (Boul et al., 1978). With prior information on the properties of these cyclic horizons, hazardous areas could be identified and acccidents avoided.

Agronomic uses of Vertisols vary widely, depending on the climate. The high clay content and associated soil properties namely, slow permeability of these soils when wet, makes them desirable for areas that require retention of surface water. Field moisture conditions, drainage conditions and patterns of vegetation indicate that the maximum oscillation between wet and dry conditions is manifested in microdepressions that retain moisture for longer periods and microknolls which dry out faster (as evidenced by moisture stress). Since this equation can help locate the lows and highs of the cyclic horizons, a farmer perhaps can use this information to assist in making management decisions. Vertisols are capable of tilting large trees. Not surprisingly, few, if any, commercial forests are found on Vertisols (Soil Surv. Staff, 1975; Buol et al., 1978). Many engineering problems also are associated with these soils. Highways, buildings, fences, pipelines, and utility lines are moved and distorted by the shrinking and swelling of these soils. Prior knowledge about the highs and lows of the cyclic pedons may help engineers and foresters plan their programs and avoid mishaps.

Conclusions

A methematical equation was developed to estimate the linear distance between microknolls and microlows in Vertisols. The equation:

$$LD\ (\text{cm}) = 200/Y(2500 + bY)^{1/2}$$

may eliminate excavating a large soil pit and thus save time and resources. With increasing awareness about the tremendous crop-supporting capacity of Vertisols in terms of their nutrient and moisture supplying power, it is essential to have more knowledge of surface features of these soils. The equation seems to be a promising new method of identifing the locations of microdepressions and microknolls to better manage Vertisols for agricultural and nonagricultural purposes.

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